which is the required formula.[1]

(*b*) Since a and d are positive acute angles, the last term cannot become negative and is zero only when $x = 0$. Therefore d will have a minimum value when $x = 0$, so that $r = r' = \frac{1}{2}a$, and $i = i'$, and the complete ray is symmetrically situated with respect to the prism.

(*c*) For this minimum value of d, say d_0, we have the classical laboratory formula

$$n = \frac{\sin \frac{1}{2}(a + d_0)}{\sin \frac{1}{2}a}.$$

2879 [1921, 89]. Proposed by E. J. OGLESBY, Washington Square College.

Given the values of $U_{5:9}$, $U_{5:10}$, $U_{5:11}$, $U_{6:9}$, $U_{6:10}$, $U_{6:11}$, $U_{7:9}$, $U_{7:10}$, $U_{7:11}$ where $U_{h:k} = \sqrt{hk}$, find the value of $U_{6\cdot 2:9\cdot 3}$ by interpolation.

Solution by J. B. Reynolds, Lehigh University.

From the first three given terms we find by the method of differences[2] $U_{5:9.3} = 6.8190$, from the next three $U_{6:9.3} = 7.4698$ and from the last three $U_{7:9.3} = 8.0685$. Using these three values then as a series by the same method we find $U_{6.2:9.3} = 7.5937$.

Also solved by H. N. Carleton.

2881 [1921, 89]. Proposed by E. B. ESCOTT, Oak Park, Ill.

If, in the polynomial $X^3 - 2$, we substitute $x^2 + x - 4$ for X, the given expression can be factored, that is, $X^3 - 2 \equiv (x^3 + 3x^2 - 3x - 11)(x^3 - 6x + 6)$. Find a substitution for X so that the polynomial $X^3 + pX^2 + qX + r$ may be factored.

Solution by the Proposer.

Let

$$X^3 + pX^2 + qX + r = (X - b)(X + c)^2 - a^2(X - d)^2. \quad (1)$$

Expanding the second member and equating coefficients of like powers of X, we get

$$a^2 + b - 2c + p = 0, \qquad 2a^2d - 2bc + c^2 - q = 0, \qquad \text{and} \qquad a^2d^2 + bc^2 + r = 0.$$

Eliminating b and d and solving for a^2 we get

$$a^2 = \frac{(3c^2 - 2pc + q)^2}{4(c^3 - pc^2 + qc - r)}.$$

Therefore, it is necessary and sufficient that [3]

$$c^3 - pc^2 + qc - r = n^2,$$

and we get by substitution

$$a = \frac{3c^2 - 2pc + q}{2n}, \qquad d = -\frac{c^3 - qc + 2r}{3c^2 - 2pc + q}, \qquad \text{and} \qquad b = -a^2 + 2c - p.$$

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[1] We might say

$$\begin{aligned}\frac{1}{n^2} &= \frac{\sin^2 \frac{1}{2}a \cos^2 x}{\sin^2 \frac{1}{2}(a + d)} + \frac{\cos^2 \frac{1}{2}a \sin^2 x}{\cos^2 \frac{1}{2}(a + d)} \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \left[\frac{\cos^2 \frac{1}{2}a}{\cos^2 \frac{1}{2}(a + d)} - \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)}\right]\sin^2 x \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \frac{4 \sin \frac{1}{2}d \sin (a + \frac{1}{2}d) \sin^2 x}{\sin^2 (a + d)}\end{aligned}$$

and use this formula for (*b*) and (*c*) instead of the formula in the text, d being a minimum when $[\sin^2 \frac{1}{2}a]/[\sin^2 \frac{1}{2}(a + d)]$ is a maximum.—Editors.

[2] See *1921*, 330.

[3] Thus it seems to be a necessary part of the hypothesis that there is a rational number $-c$ that will make the given polynomial equal to minus the square of a rational number.

Letting f denote the polynomial, if $f(-c) = -n^2$ we can write

$$f(X) = (X + c)^2\left[X - 2c + p + \left(\frac{f'(-c)}{2n}\right)^2\right] - \left[\frac{f'(-c)}{2n}(X + c) - n\right]^2,$$

which we can make the difference of two squares by putting $X - 2c + p + \left(\frac{f'(-c)}{2n}\right)^2 = (x + t)^2$.

—Editors.